HARROW COLLEGE & UXBRIDGE COLLEGE (HCUC)

A-LEVEL MATHEMATICS

STUDY PACK

HCUC

A merger between Uxbridge College and Harrow College

Harrow COLLEGE

UXBRIDGE COLLEGE

A-Level Mathematics Transition Activities

UXBRIDGE COLLEGE

The purpose of this activity is to aid the preparation of potential AS Mathematics students for advanced level study and make the transition from GCSE study as smooth as possible. The activities should be completed to the best of your ability and they will give you the opportunity to start to showcase your talent for Mathematics.

AS Maths has been designed to follow on from the new, more challenging GCSE in Maths, as a course for students who enjoy and are successful in Higher tier GCSE algebra topics.

Studying A Levels is very different from taking GCSEs. On all courses, the pace will be much faster than what you are used to at school, meaning that homework is very important, just so you can keep up with the course. The amount of homework you have to do will be much more than at GCSE level, and if you are stuck you will be expected to actively seek out help so that you can hand in completely finished work, for instance by sending your teacher an email or arranging to see them during a study period, lunchtime or before or after college.

You will need to get into a lot of good habits to become a successful A Level Maths student.

Conal Ruddy & Sella Jeyakumar
Uxbridge College

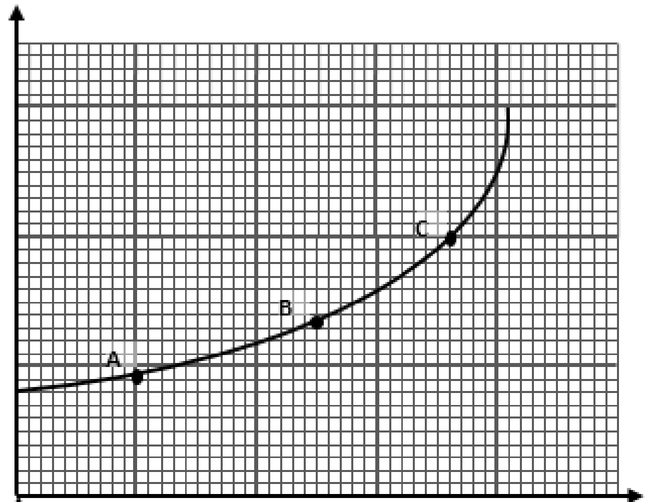
What will I study in AS Maths?

Just over two-thirds of the course is algebra, with the remaining topics split between statistics and mechanics. An outline of the topics solved is

Algebra Topics	Statistics Topics	Mechanics Topics
• Surds and indices • Quadratic functions • Simultaneous equations • Equations of lines, curves and circles • The Factor Theorem • Binomial Expansions • Transformations of graphs • Trigonometry and trigonometric equations • Differentiation and integration • Logarithms • Exponential models • Vectors	• Sampling • Measure of central tendency and spread • Histograms, scatter diagrams, box and whisker diagrams • Probability • The binomial distribution • Hypothesis testsing	• Travel graphs • Speed, velocity, time, displacement and constant acceleration • Variable acceleration • Forces

Topic 1: Gradients of Curves

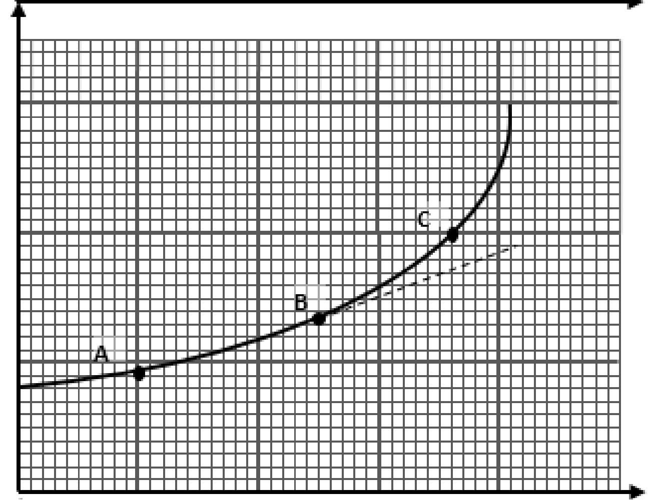
Graphs are an important part of determining a rate of change. At this level, you should be able to find the gradient to a curve.



The curve, shown, varies in steepness.

The section between B and C is steeper than that between A and B.

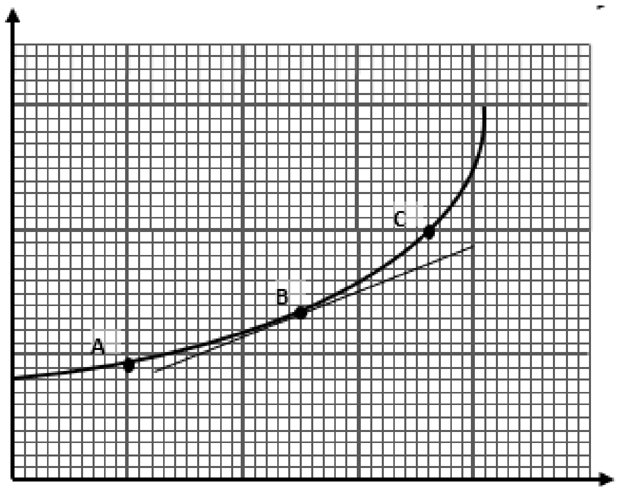
By definition gradient is a measure of steepness, the gradient of the curve changes throughout.



As we move along the curve from A to B the gradient increases.

If, at B, we continue straight ahead (along the dotted line) the gradient stops increasing and has the same value as at B.

The gradient of the curve at B is then the gradient of the dotted line.



The dotted line at B may be extended backwards as shown here.

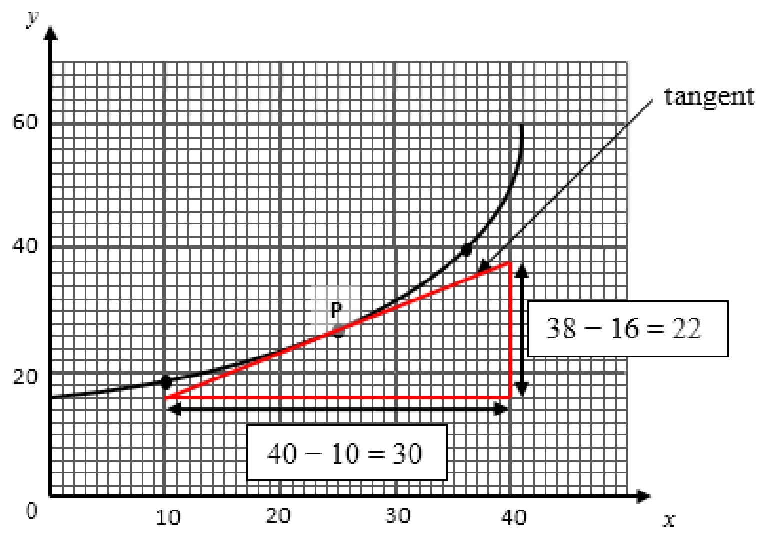
The dotted line becomes a solid line.

This line **touches** the curve at B and is called the **tangent** at B.

In summary,

The gradient of a curve at a point is the gradient of the tangent to the curve at that point.

How do we find the gradient of this curve at the point P?



To find the gradient at point P:

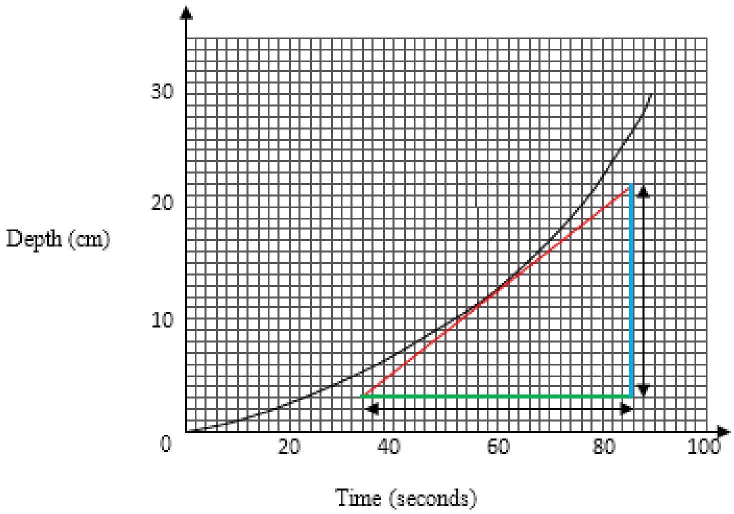
- Draw a tangent to the point P
- Use the tangent (hypotenuse) to complete a right-angled triangle

Hence,

$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{22}{30} = 0.733$$

EXAMPLE 1

Asha poured liquid into a container.
She recorded her results and then drew a graph as shown.



(a) Work out the gradient of this curve when $t = 60$ seconds.

(b) Interpret your answer to part (a).

First draw a tangent at $t = 60$ seconds (shown in **red** on the graph).

Then draw a right-angled triangle (shown in **blue** and **green** on the diagram).

Try to draw the triangle so that it goes through easy points to read on the graph.

(a)

Change in depth (y) (in blue) = 22 − 3 = 19 ← Read the y-coordinates on your triangle

Change in time (x) (in green) = 86 − 34 = 52 ← Read the x-coordinates on your triangle

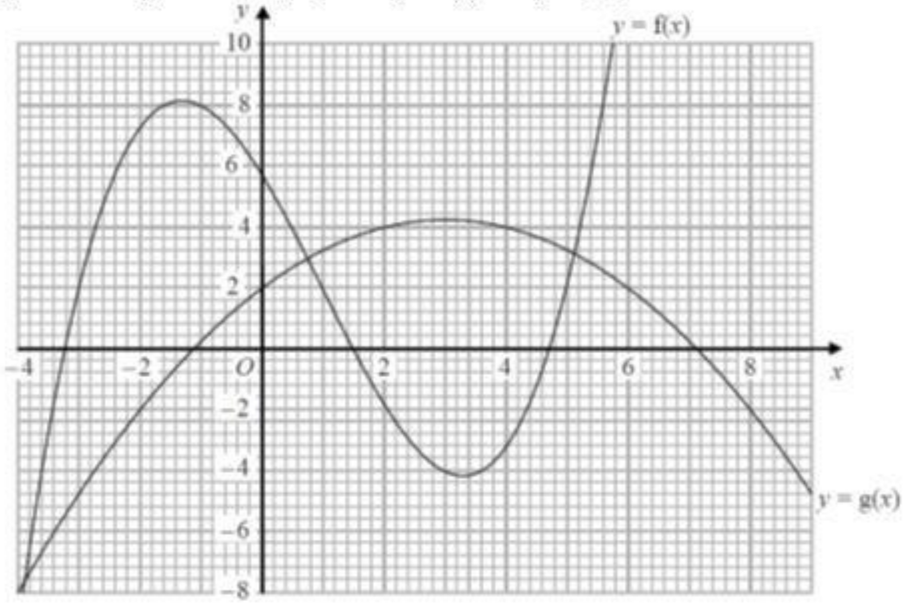
$$\text{gradient} = \frac{\text{chnagein depth}}{\text{changein time}} = \frac{19}{52} = 0.365$$

(b) When $t = 60$ seconds the depth of the water is increasing at a rate of 0.365 cm per second

← Vertical distance = cm
Horizontal distance = seconds
cm/sec = cm per second
Depth is increasing as positive gradient

EXERCISE:

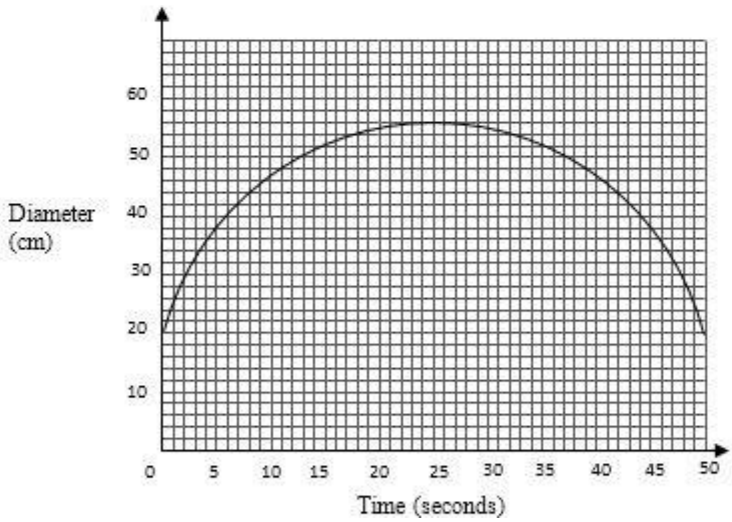
1. The diagram shows parts of the graphs of $y = f(x)$ and $y = g(x)$.



(a) Write down the value of x where the gradient of the curve $y = g(x)$ is zero.

(b) Calculate an estimate for the gradient of the curve $y = f(x)$ at the point on the curve where $x = 4$

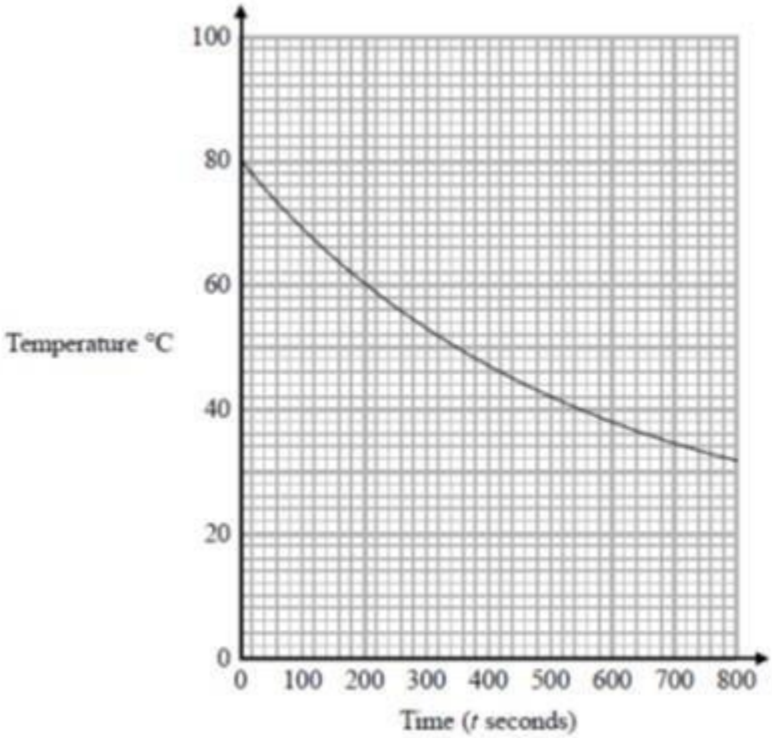
2. A fish bowl is being filled with water.
The graph shows how the diameter of the surface of the water changes with time.



(a) Work out the gradient when the time is 10 seconds.

(b) Give an interpretation of the gradient.

3. The graph gives information about the variation in the temperature, in °C, of an amount of water that is allowed to cool from 80 °C.



(a) Work out the rate of decrease of temperature at $t = 400$

(b) Work out the average rate of decrease of the temperature of the water between $t = 0$ and $t = 800$.

The instantaneous rate of decrease of the temperature of the water at time T seconds is equal to the average rate of decrease of the temperature of the water between $t = 0$ and $t = 800$

(c) Find an estimate for the value of T.
You must show how you got your answer.

Topic 2: Turning Points Using Completing the Square

You should already be able to express a quadratic equation in the form $a(x + b)^2 + c$ by completing the square.

e.g. $x^2 - 6x + 3 = (x - 3)^2 - 9 + 3 = (x - 3)^2 - 6$

e.g. $3x^2 + 6x + 5 = 3[x^2 + 2x] + 5 = 3[(x + 1)^2 - 1] + 5 = 3(x + 1)^2 + 2$

We are now going to deduce the turning points of a quadratic function after completing the square.

EXAMPLE 1

Given $y = x^2 + 6x - 5$, by writing it in the form $y = (x + a)^2 + b$, where a and b are integers, write down the coordinates of the turning point of the curve. Hence sketch the curve.

$y = x^2 + 6x - 5$

$= (x + 3)^2 - 9 - 5$

$= (x + 3)^2 - 14$

Remember to halve the coefficient of x

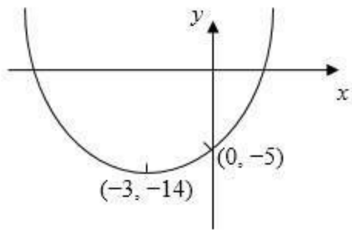
and subtract $(-3)^2$ to compensate

The turning point occurs when $(x + 3)^2 = 0$, i.e. when $x = -3$
When $x = -3$, $y = (-3 + 3)^2 - 14 = 0 - 14 = -14$

So the coordinates of the turning point is $(-3, -14)$

The graph $y = x^2 + 6x - 5$ cuts the y-axis when $x = 0$, i.e. $y = -5$

Sketch:



When $y = (x + a)^2 + b$ then the coordinates of the turning point is $(-a, b)$.

The minimum or maximum value of y is b.

EXAMPLE 2

Given that the minimum turning point of a quadratic curve is $(1, -6)$, find an equation of the curve in the form $y = x^2 + ax + b$. Hence sketch the curve.

$y = (x - 1)^2 - 6$

$= (x^2 - x - x + 1) - 6$

$= x^2 - 2x - 5$

If the minimum is when $x = 1$, we know we have $(x - 1)^2$

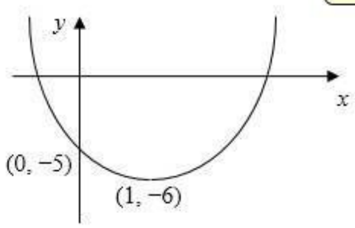
If the minimum is when $y = -6$, we know we have $(...)^2 - 6$

An equation of the curve is $y = x^2 - 2x - 5$

The graph cuts the y-axis when $x = 0$, i.e. at $y = -5$

Sketch:

It is a minimum turning point so the shape is ◡



NOTE: There are other possible equations as, for example $y = 4(x - 1)^2 - 6$ also has a turning point of $(1, -6)$. If it was a maximum turning point then the coefficient of x^2 would be negative.

EXERCISE:

1. By writing the following in the form $y = (x + a)^2 + b$, where a and b are integers, write down the coordinates of the turning point of the curve. Hence sketch the curve.

(a) $y = x^2 - 8x + 20$
(b) $y = x^2 - 10x - 1$
(c) $y = x^2 + 4x - 6$
(d) $y = 2x^2 - 12x + 8$
(e) $y = -x^2 + 6x + 10$
(f) $y = 5 - 2x - x^2$

2. Given the following minimum turning points of quadratic curves, find an equation of the curve in the form $y = x^2 + ax + b$. Hence sketch each curve.

(a) $(2, -3)$
(b) $(-4, 1)$
(c) $(-1, 5)$
(d) $(3, -12)$
(e) $(1, -7)$
(f) $(-4, -1)$

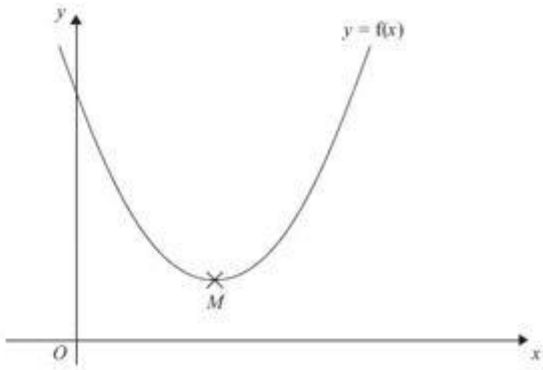
3. Find the maximum or minimum value of the following curves and sketch each curve.

(a) $y = x^2 + 4x + 2$
(b) $y = 1 - 6x - x^2$
(c) $y = -x^2 + 2x - 3$
(d) $y = x^2 - 8x + 8$
(e) $y = x^2 - 3x - 1$
(f) $y = -3x^2 + 12x - 9$

4. The expression $x^2 - 3x + 8$ can be written in the form $(x - a)^2 + b$ for all values of x.

(i) Find the value of a and the value of b.

The equation of a curve is $y = f(x)$ where $f(x) = x^2 - 3x + 8$
The diagram shows part of a sketch of the graph of $y = f(x)$.



The minimum point of the curve is M.

(ii) Write down the coordinates of M.

5. (i) Sketch the graph of $f(x) = x^2 - 6x + 10$, showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.

   (ii) Hence, or otherwise, determine whether $f(x) - 3 = 0$ has any real roots.
   Give reasons for your answer.

*6. The minimum point of a quadratic curve is $(1, -4)$. The curve cuts the y-axis at -1.
Show that the equation of the curve is $y = 3x^2 - 6x - 1$

*7. The maximum point of a quadratic curve is $(-2, -5)$. The curve cuts the y-axis at -13.
Find the equation of the curve. Give your answer in the form $ax^2 + bx + c$.

* = extension

Topic 3: Simultaneous Equations

Q1.

Solve the simultaneous equations

$$5x + 2y = 11$$
$$4x - 3y = 18$$

x = ...

y = ...

(4)

Q2.

Solve the simultaneous equations

$$4x + y = 25$$
$$x - 3y = 16$$

x = ...
y = ...

(3)

Q3.

Solve the simultaneous equations

$$x^2 + y^2 = 9$$
$$x + y = 2$$

Give your answers correct to 2 decimal places.

x = ... y = ...
or x = ... y = ...

(6)

Q4.
Find the coordinates of intersection of the circle $x^2 + y^2 = 25$ and the straight line $y = 2x + 5$.

(6)

THE END

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